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RESEARCH ARTICLE

The capacity of multi-channel multi-interface wireless networks with multi-packet reception and directional antenna[†]

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ABSTRACT

The capacity of wireless networks can be improved by the use of multi-channel multi-interface (MCMI), multi-packet reception (MPR), and directional antenna (DA). MCMI can provide the concurrent transmission in different channels for each node with multiple interfaces; MPR offers an increased number of concurrent transmissions on the same channel; DA can be more effective than omni-DA by reducing interference and increasing spatial reuse. This paper explores the capacity of wireless networks that integrate MCMI, MPR, and DA technologies. Unlike some previous research, which only employed one or two of the aforementioned technologies to improve the capacity of networks, this research captures the capacity bound of the networks with all the aforementioned technologies in arbitrary and random wireless networks. The research shows that such three-technology networks can achieve at most $\frac{2\pi}{\theta}\sqrt{k}$ capacity gain in arbitrary networks and $\left(\frac{2\pi}{\theta}\right)^2 k$ capacity gain in random networks compared with MCMI wireless networks without DA and MPR. The paper also explored and analyzed the impact on the network capacity gain with different $\frac{c}{m}$, θ, and k-MPR ability.

KEYWORDS

network capacity; multi-channel multi-interface; multi-packet reception; directional antenna

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1. INTRODUCTION

It is commonly accepted that the presence of multi-channel multi-interface (MCMI)-enabled devices is highly possible in the near future. A network based on multi-channel as compared with one single common channel, enables the full use of the network resources. The rapid growth of the IEEE 802.11 technology [1], which is widely used in wireless communications, has led to the sharp price decline of the MCMI devices. This price decline has resulted in wider acceptance and use of these devices. When using either the traditional medium access control (MAC) or the new MCMI technology, simultaneous transmissions may cause collisions, and the collision packets are retransmitted later. Ghez *et.al* proposed multi-packet reception (MPR) technology [2,3], which makes it possible for the node to receive multiple packets simultaneously to solve this problem. This technology could increase the number of concurrent transmissions in the network and thus effectively improves network capacity and reduces the delay of received packets. In addition, recent literature [4–7] found that using directional antennas (DAs) instead of omni-DAs in wireless networks can greatly improve the network capacity, because DAs can reduce the collision area. In [8], a countermeasure is proposed by using DAs to prevent wormhole attacks. As demonstrated in a previous research, there are a series of benefits in using DAs.

The combined using of MCMI, MPR, and DA is more efficient than using just one or two of these three techniques. It is important to note that although benefits of

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such combinations do exist in the majority of cases, caution must always be exercised when using any combination under any condition. For example, in the region where the density of mobile users is relatively low, the distribution of users is sparse and the demand of network resources is not very high. Thus, the number of base stations is generally small and the density is low. Under this circumstance, omni-DA would be a better choice. Yet, the bound of network capacity which uses such combined technology is required necessarily by further research for MCMI networks with MPR and DA, and it motivates us to capture the bound of capacity in such networks.

This paper studied the capacity of a network that combined the three technologies: MCMI, MPR, and DA. In this network, each node is equipped with multiple interfaces and each interface is associated with one DA that can operate on different channels. Considering MPR nodes' decode ability, the number of decode transmissions is limited by one node at a time. The assumption is that a wireless interface could decode at most k, $k > 1$, concurrent transmissions within its receiving range and each node could use m interfaces, each of which can work on one of the total c channels. In addition, the beamwidth of all DAs is identical θ. We call such networks (m, c, θ, k) wireless networks. Similarly, MCMI networks with omni-DAs and no-MPR are called $(m, c, 2\pi, 1)$ wireless networks. To the best of our knowledge, there is no theoretical analysis on the capacity of such (m, c, θ, k) wireless networks. This paper focused on finding the capacity bound for a (m, c, θ, k) network in both arbitrary and random networks and on exploring the benefits of such networks.

The remainder of the paper is organized as follows. We summarize the related work in Section 2. The network model, antenna model, and main results are described in Section 3, and the proof for the results of arbitrary networks is presented in Section 4. Section 5 presents the proof for the results of random networks. Finally, we discuss the results in Section 6 and conclude the paper in Section 7.

2. RELATED WORK

The capacity of the network is essential to network research regardless of any layer on which those researchers focus, because it provides the theoretical bound reflecting the capacity of transmission. Literature [9] is known as the milestone of research on network capacity. It established a classic network model of wireless networks and gave a clear definition of network capacity. It demonstrated the capacity scales as $\Theta(W\sqrt{n})$ bit-m/s in an arbitrary network and the network capacity scales as $\Theta\left(W\sqrt{\frac{n}{\log n}}\right)$ bits/s in a random network, and it also considered multi-channel conditions; the research proved that the capacity of a network with a single channel and one interface per node is equal to the capacity of a network with c channels and $m = c$ interfaces per node. However, the research did not capture the impact on capacity by using fewer than c interfaces per node. Kyasanur and Vaidya [10] explored the capacity of MCMI networks, especially when $c \le m$. The results showed that the capacity is dependent on the ratio of $\frac{c}{m}$, not on the exact value of either c or m.

Furthermore, the current research trend is to improve the network capacity by using MPR. Garcia-Luna-Aceves *et.al* [11] have shown that a 3D random MPR-based wireless network has a capacity gain of $\Theta(\log n)$. M.F Guo *et.al* [12,13] addressed the capacity of single channel single interface (SCSI) and MCMI networks by using k-MPR, respectively. For MCMI networks, the network capacity degrades as the ratio of $\frac{c}{m}$ increases in both arbitrary and random networks. However, there is no capacity degradation as the ratio of $\frac{c}{m}$ increases when $\frac{c}{m} = O(\log(n))$ and $k = O\left(\sqrt{\frac{m\log(n)}{c}}\right)$ in random networks.

Besides, the use of DAs is a key technique to reduce the interference area, Yi *et.al* [7] have shown that the capacity gain is $\frac{2\pi}{\theta}$ in arbitrary networks and $\frac{4\pi^2}{\theta^2}$ in random networks for SCSI networks. Literature [14] showed that the capacity gain using the hybrid antenna mode is $\sqrt{\frac{\theta^2}{\theta^2+s(4\pi^2-\theta^2)}}$ over the antenna with the simplified model for MCMI networks. H. Dai *et.al* [15] studied the throughput capacity of multi-channel wireless networks with a DA. The research showed that the use of DAs achieves higher capacity than omni-DAs when $\frac{c}{m} = O(n)$ in arbitrary networks and $\frac{c}{m} = O\left(n\left(\frac{\log\log n}{\log n}\right)^2\right)$ in random networks.

In view of the survey of related work, this paper is the first work to study the capacity of 2D MPR-based MCMI wireless networks by using DAs. The goal of this work is to study the impact of the ratio of $\frac{c}{m}$, beamwidth of DA, and k-MPR ability in a network which integrates three technologies.

3. MODELS AND MAIN RESULTS

3.1. Network model

This paper considers the setting on a planar disk of unit area. There are c non-overlapping channels and each node is equipped with m wireless DAs, $1 \le m \le c$, and the beamwidth of DA is identical θ. The assumption is that a wireless interface could decode at most k, $k > 1$, concurrent transmissions within its receiving range. We also assume that c channels can support a data rate of W bits/s and any one of c channels can support $\frac{W}{c}$ bits/s. In addition, there are n nodes located in the unit area and the network transports $\lambda n T$ bits over T seconds.

Similar as [9], two network models are established for arbitrary network and random network in order to capture the capacity of (m, c, θ, k) wireless networks. $X \Leftrightarrow Y$ is used to represent that node X points its antenna beam towards node Y and vice versa.

3.1.1. Arbitrary network.

In the arbitrary (m, c, θ, k) network setting, each node has an arbitrarily chosen destination, to which it could send traffic at an arbitrary rate. Each node can choose arbitrary power level for each transmission. Under these assumptions, we give the protocol interference model for arbitrary (m, c, θ, k) wireless networks as follows.

Suppose k nodes, $\{X_i | 1 \le i \le k\}$, transmit packets to node X_j simultaneously. Packets can be successfully received by node X_j if

$$|X_q - X_j| \ge max_{1 \le i \le k}(1+\Delta)|X_i - X_j| \; and \; X_i \Leftrightarrow X_j$$

for every other node X_q , where $X_q \Leftrightarrow X_j$, simultaneously transmitting to other node over the same channel. The quantity $\Delta > 0$ models a guard zone that prevents interfering nodes from transmitting on the same channel at the same time.

3.1.2. Random network.

In a random (m, c, θ, k) wireless network, nodes are randomly located, independently and uniformly distributed. Each node has one flow to a randomly chosen destination, to which it wishes to send at $\lambda(n)$ bits/s. Each node employs the same receiving range $r(n)$. Under these assumptions, we give the protocol interference model for random (m, c, θ, k) wireless networks as follow.

Suppose k nodes, $\{X_i | 1 \le i \le k\}$, transmit packets to node X_j simultaneously. Packets can be successfully received by node X_j if

(1) $max_{1 \le i \le k}|X_i - X_j| \le r(n) \; and \; X_i \Leftrightarrow X_j$

(2) $|X_q - X_j| \ge (1+\Delta)r(n)$

for every other node X_q , where $X_q \Leftrightarrow X_j$, simultaneously transmitting over the same channel.

3.2. Antenna model

Antenna systems may be broadly classified into omni-directional and DA systems. In omni-directional mode, a node is capable of transmitting signals to all directions, and signals can also interfere with other unrelated nodes. In the directional mode, a node can point its beam towards a specified direction. The direction in which the main lobe should point for a given transmission is specified to the antenna by the MAC protocol.

In the DA model of this paper, we adopt the unidirectional pattern [16] and extend it to a circle sector to model the DA pattern with radius r and angle θ. In unidirectional pattern, the main lobe is the direction of maximum radiation or reception. In addition to the main lobe, there are also sidelobes and backlobes. It is well-known that good antenna designs attempt to minimize them, because these lobes represent lost energy. This paper assumes that the DA gain is within the main lobe, and the gain outside the main lobe is assumed to be zero. As shown in Figure 1, in our antenna pattern, radius r represents the transmission range or receiving range, and angle θ is the beamwidth of antennas. In Figure 1(a), two nodes' antenna point towards each other, $X_i \Leftrightarrow X_j$. Two nodes are located in each other's interference area, so they can build a successfully wireless link. As shown in Figure 1(b), the beam of node X_j is not pointed towards X_i in spite that X_i's beam

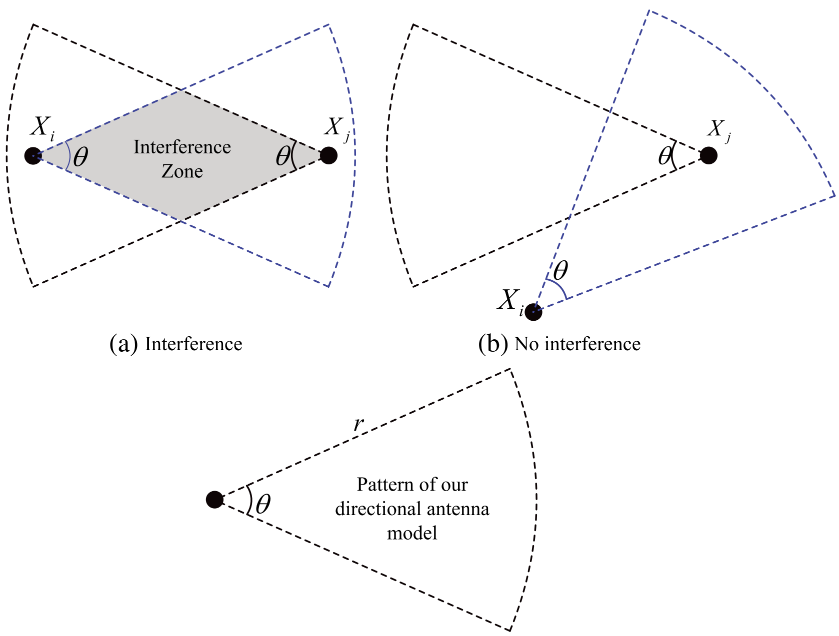

Figure 1. Directional antenna pattern and interference condition.

points towards X_j; therefore, X_j cannot receive packets successfully from X_i and vice versa.

3.3. Main results

The results of this paper are presented as follows. G^1, G^2 are used to denote capacity gain of $\frac{2\pi}{\theta}$, $(\frac{2\pi}{\theta})^2$ over $(m, c, 2\pi, k)$ wireless networks, respectively, and G^0 is used to represent no capacity gain. This paper uses Knuth's notation [17] to represent the following asymptotic bounds:

(1) $f(n) = O(g(n))$ denotes that there exists some constant d and integer N, such that $f(n) \le dg(n)$ for $n > N$.
(2) $f(n) = \Omega(g(n))$ denotes $g(n) = O(f(n))$.
(3) $f(n) = \Theta(g(n))$ denotes $f(n) = O(g(n))$ and $g(n) = O(f(n))$.
(4) $MIN_O(f(n), g(n))$ is equal to $f(n)$, if $f(n) = O(g(n))$; else, is equal to $g(n)$.

For the results of this paper, θ is used to denote $\frac{\theta}{2\pi}$, because $\frac{1}{2\pi}$ can be regarded as a constant.

3.3.1. Arbitrary network.

The network capacity of arbitrary networks is measured in terms of 'bit-m/s' (used in [9]). The upper bound and lower bound for the capacity of arbitrary networks match exactly. The network capacity is presented as follows:

(1) When $\frac{kc}{m} = O(n\theta^2)$, network capacity is $\Theta\left(\frac{W}{\theta}\sqrt{\frac{kmn}{c}}\right)$ bit-m/s, whose gain is G^1.
(2) When $\frac{kc}{m} = \Omega(n\theta^2)$, network capacity is $\Theta\left(\frac{Wmn}{c}\right)$ bit-m/s.

The capacity of arbitrary networks is illustrated in Figure 2. The capacity of arbitrary networks with different values of k is also illustrated in Figure 3 to make these results more clearly.

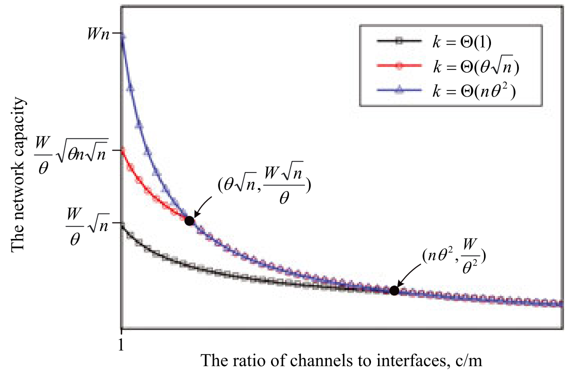

Figure 3. The capacity of arbitrary (m, c, θ, k) wireless networks with different values of k (figure is not to scale).

3.3.2. Random network.

The network capacity of random networks is measured in terms of 'bits/s'. The upper bound and lower bound for the network capacity are presented as follows:

(1) Upper bound

① . When $k = O\left(\frac{\log n}{\log\log n}\right)$

i. When $\frac{c}{m} = O\left(\frac{\theta^2 \log n}{k}\right)$, $\lambda n\overline{L} = O\left(\frac{Wk}{\theta^2}\sqrt{\frac{n}{\log n}}\right)$ bits/s, whose gain is G^2.
ii. When $\frac{c}{m} = \Omega\left(\frac{\theta^2 \log n}{k}\right)$ and $\frac{c}{m} = O\left(\theta^2 nk\left(\frac{\log\log n}{\log n}\right)^2\right)$, $\lambda n\overline{L} = O\left(\frac{W}{\theta}\sqrt{\frac{kmn}{c}}\right)$ bits/s, whose gain is G^1.
iii. When $\frac{c}{m} = \Omega\left(\theta^2 nk\left(\frac{\log\log n}{\log n}\right)^2\right)$, $\lambda n\overline{L} = O\left(\frac{Wmnk \log\log n}{c\log n}\right)$ bits/s.

② . When $k = \Omega\left(\frac{\log n}{\log\log n}\right)$ and $k = O(n)$

i. When $\frac{c}{m} = O(\theta^2 \log\log n)$, $\lambda n\overline{L} = O\left(\frac{W}{\theta^2}\sqrt{\frac{nk}{\log\log n}}\right)$ bits/s, whose gain is G^2.

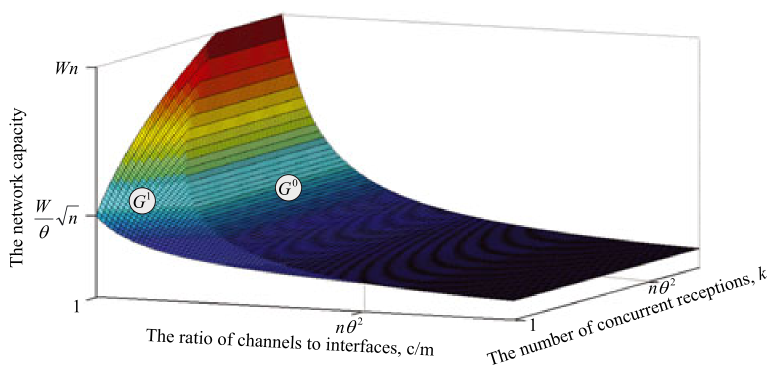

Figure 2. The capacity of arbitrary (m, c, θ, k) wireless networks (figure is not to scale).

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ii. When $\frac{c}{m} = \Omega\left(\theta^2 \log\log n\right)$, $\frac{c}{m} = O\left(\frac{n\theta^2}{k}\right)$ and $k = O\left(\frac{n}{\log\log n}\right)$, $\lambda n\overline{L} = O\left(\frac{W}{\theta}\sqrt{\frac{kmn}{c}}\right)$ bits/s, whose gain is G^1.

iii. When $\frac{c}{m} = \Omega\left(\frac{n\theta^2}{k}\right)$, $\lambda n\overline{L} = O\left(\frac{Wmn}{c}\right)$.

③. When $k = \Omega(n)$

i. When $\frac{c}{m} = O(\theta^2\sqrt{\log\log n})$, $\lambda n\overline{L} = O\left(\frac{Wn}{\theta^2\sqrt{\log\log n}}\right)$ bits/s, whose gain is G^2.

ii. When $\frac{c}{m} = \Omega(\theta^2\sqrt{\log\log n})$, $\lambda n\overline{L} = O\left(\frac{Wmn}{c}\right)$ bits/s.

The capacity is illustrated in three figures with different ranges of k (Figures 4, 6, and 8) to clearly comprehend the upper bound for capacity of random networks. The upper bound for capacity of random networks with different values of k is also illustrated in three figures (Figures 5, 7 and 9) to depict the results with different ranges of k, respectively.

(2) Lower bound

①. When $\frac{kc}{m} = O(\theta^2 \log n)$

i. When $\frac{c}{m} = O\left(\theta^2\sqrt{\frac{(\log n)^3}{n}}\right)$ and $k = O\left(\sqrt{\frac{n}{\log n}}\right)$, $\lambda n\overline{L} = \Omega\left(\frac{Wk}{\theta^2}\sqrt{\frac{n}{\log n}}\right)$ bits/s.

ii. When $\frac{c}{m} = O\left(\theta^2\sqrt{\frac{(\log n)^3}{n}}\right)$ and $k = \Omega\left(\sqrt{\frac{n}{\log n}}\right)$, $\lambda n\overline{L} = \Omega\left(\frac{Wn}{\theta^2 \log n}\right)$ bits/s.

iii. When $\frac{c}{m} = \Omega\left(\theta^2\sqrt{\frac{(\log n)^3}{n}}\right)$ and $k = O\left(\frac{\theta^2 m \log n}{c}\right)$, $\lambda n\overline{L} = \Omega\left(\frac{Wk}{\theta^2}\sqrt{\frac{n}{\log n}}\right)$ bits/s.

②. When $\frac{kc}{m} = \Omega(\theta^2 \log n)$ and also $O\left(\theta^2 n\left(\frac{\log\log n}{\log n}\right)^2\right)$, $\lambda n\overline{L} = \Omega\left(\frac{W}{\theta}\sqrt{\frac{mn}{c}}\right)$ bits/s.

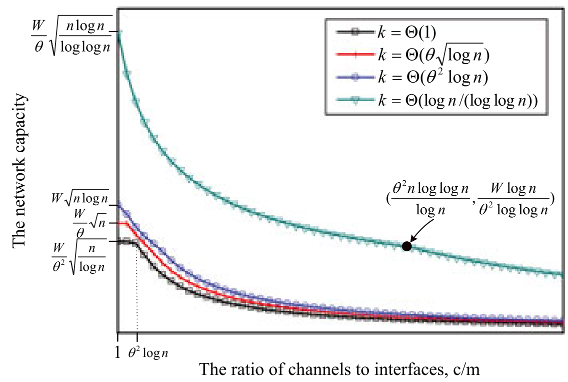

Figure 5. The upper bound for the capacity of random (m, c, θ, k) wireless networks with different values of k ($k = O\left(\frac{\log n}{\log\log n}\right)$, and figure is not to scale).

③. When $\frac{kc}{m} = \Omega\left(\theta^2 n\left(\frac{\log\log n}{\log n}\right)^2\right)$, $\lambda n\overline{L} = \Omega\left(\frac{Wmn\log\log n}{c\log n}\right)$ bits/s.

4. CAPACITY OF ARBITRARY NETWORK

4.1. Upper bound

In an arbitrary (m, c, θ, k) wireless network, we suppose that the whole network transmits λnT over T seconds. The average distance between the source and destination of a bit is $\overline{L}$. Hence, a capacity of $\lambda n\overline{L}$ is achieved.

Lemma 1. *Under the protocol model, the capacity of arbitrary (m, c, θ, k) wireless networks is bounded as follows:*

(1) When $k = O(n)$, arbitrary (m, c, θ, k) network capacity is $O\left(\frac{W}{\theta}\sqrt{\frac{kmn}{c}}\right)$ bit-m/s.

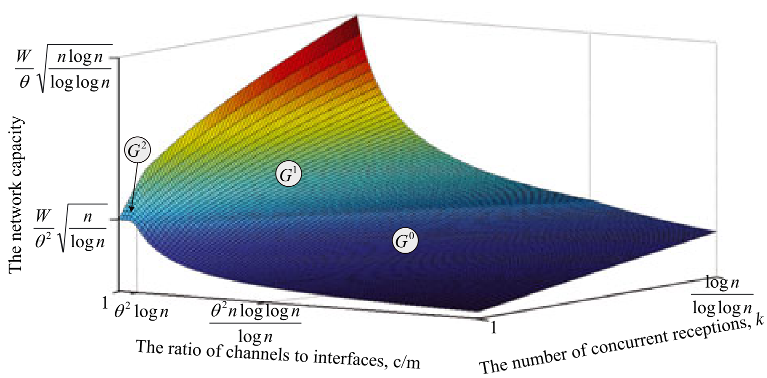

Figure 4. The upper bound for the capacity of random (m, c, θ, k) wireless networks when $k = O\left(\frac{\log n}{\log\log n}\right)$ (figure is not to scale).

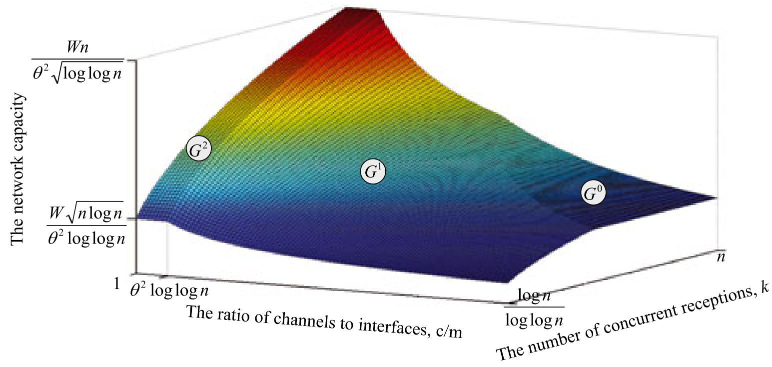

Figure 6. The upper bound for the capacity of random (m, c, θ, k) wireless networks when $k = \Omega\left(\frac{\log n}{\log\log n}\right)$ and also $k = O(n)$ (figure is not to scale).

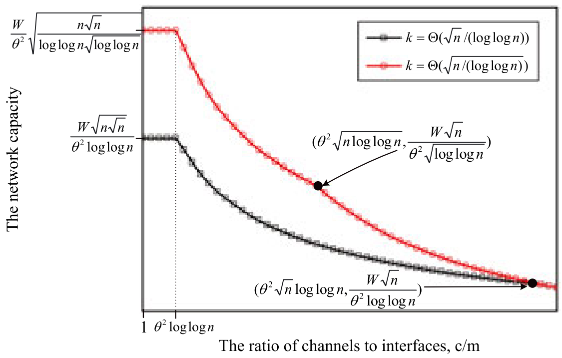

Figure 7. The upper bound for the capacity of random (m, c, θ, k) wireless networks with different values of k ($k = \Omega\left(\frac{\log n}{\log\log n}\right)$, $k = O(n)$ and figure is not to scale).

(2) When $k = \Omega(n)$, arbitrary (m, c, θ, k) network capacity is $O\left(\frac{Wn}{\theta}\sqrt{\frac{m}{c}}\right)$ bit-m/s.

Proof. When $k = O(n)$, considering bit b, $1 \leq b \leq \lambda n T$. Supposing that it moves from its source to its destination in a sequence of $h(b)$ hops, where the h-th hop traverses a distance of r_b^h. Then we have

$$\lambda n T \overline{L} \leq \sum_{b=1}^{\lambda n T}\sum_{h=1}^{h(b)} r_b^h. \qquad (1)$$

We should group the nodes into $\left(\left\lfloor\frac{n}{k+1}\right\rfloor + 1\right)$ groups to utilize the k-MPR ability. Note that we are trying to upper bound the network capacity; hence, such node grouping is only an ideal case and need not to be feasible. Each of these groups has k transmitters and one receiver. Because each node is equipped with m interfaces, the first $\left\lfloor\frac{n}{k+1}\right\rfloor$ groups totally contain at most $\left\lfloor\frac{n}{k+1}\right\rfloor mk$ transmission pairs and each of these groups contains at most mk transmission pairs. The last one group contains $m\left(n - \left\lfloor\frac{n}{k+1}\right\rfloor(k+1) - 1\right)$ transmission pairs. Consequently, there are totally at most $m\left(n - \left\lfloor\frac{n}{k+1}\right\rfloor - 1\right)$ transmissions in any time slot. Hence, we have

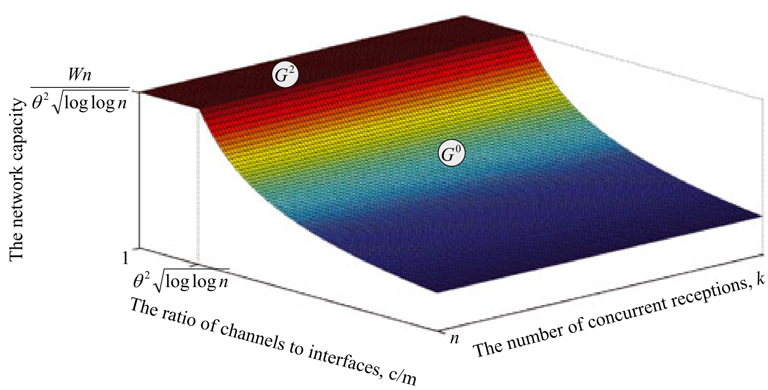

Figure 8. The upper bound for the capacity of random (m, c, θ, k) wireless networks when $k = \Omega(n)$ (figure is not to scale).

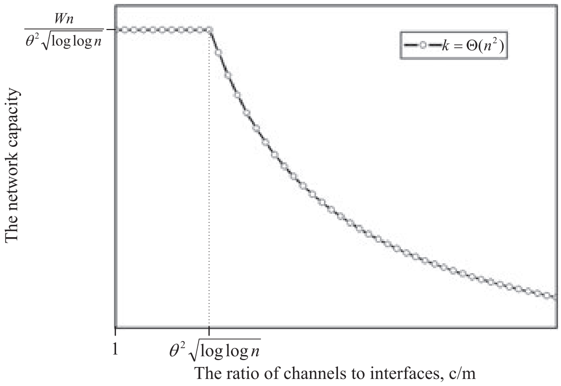

Figure 9. The upper bound for the capacity of random (m, c, θ, k) wireless networks with different values of k ($k = \Omega(n)$ and figure is not to scale).

$$H := \sum_{b=1}^{\lambda n T} h(b) \le \frac{WTm}{c}\left(n - \left\lfloor \frac{n}{k+1} \right\rfloor - 1\right) \le \frac{WTmn}{c}. \tag{2}$$

It is shown in [9] that each hop consumes a disk of radius $\frac{\Delta}{2}$ times the length of the hop around each receiver. What is more, when a node points its beam towards a receiver and the receiver is only affected by the nodes within its antenna beam. On the average, $\frac{\theta}{2\pi}$ proportion of the nodes inside the reception beam could interfere with the receiver. Thus, the conditional interference zone area is $\pi\left(\frac{\Delta}{2} r_b^h\right)^2 \left(\frac{\theta}{2\pi}\right)^2 = \frac{\Delta^2\theta^2}{16\pi}\left(r_b^h\right)^2$. Hence, the number of simultaneous transmissions $T(n)$ multiplied by $\frac{\Delta^2\theta^2}{16\pi}\left(r_b^h\right)^2$ must be less than the area of network (equal to 1).

$$T(n) \le \frac{16\pi}{\Delta^2\theta^2\left(r_b^h\right)^2} \tag{3}$$

Because the number of concurrent transmissions within the disk of radius $\frac{\Delta}{2} r_b^h$ centered at a receiver is at most k, in arbitrary (m, c, θ, k) wireless networks, we have

$$\sum_{b=1}^{\lambda n T}\sum_{h=1}^{h(b)} 1\,(\text{The } h-\text{th hop of bit } b \text{ over sub-channel } m \text{ in slot } s)\,\frac{\Delta^2\theta^2}{16\pi k}\left(r_b^h\right)^2 \le \frac{W\tau}{c}. \tag{4}$$

By summing over all the channels (which can in total potentially transport W bits), we have the constraint,

$$\sum_{b=1}^{\lambda n T}\sum_{h=1}^{h(b)} \frac{\Delta^2\theta^2}{16\pi k}\left(r_b^h\right)^2 \le WT. \tag{5}$$

The equation (5) can be rewritten as

$$\sum_{b=1}^{\lambda n T}\sum_{h=1}^{h(b)} \frac{1}{H}\left(r_b^h\right)^2 \le \frac{16\pi k W T}{\Delta^2\theta^2 H}. \tag{6}$$

Because the quadratic function is convex, we have

$$\left(\sum_{b=1}^{\lambda n T}\sum_{h=1}^{h(b)} \frac{1}{H} r_b^h\right)^2 \le \sum_{b=1}^{\lambda n T}\sum_{h=1}^{h(b)} \frac{1}{H}\left(r_b^h\right)^2. \tag{7}$$

Combining (6) and (7) yields

$$\sum_{b=1}^{\lambda n T}\sum_{h=1}^{h(b)} \frac{1}{H} r_b^h \le \sqrt{\frac{16\pi k W T}{\Delta^2\theta^2 H}}. \tag{8}$$

Then, substituting (2) in (7) gives

$$\sum_{b=1}^{\lambda n T}\sum_{h=1}^{h(b)} r_b^h \le \frac{WT}{\Delta\theta}\sqrt{\frac{16\pi k m n}{c}}. \tag{9}$$

Finally, we substitute (1) in (9) and obtain

$$\lambda n \overline{L} \le \frac{W}{\Delta\theta}\sqrt{\frac{16\pi k m n}{c}}. \tag{10}$$

When $k = \Omega(n)$, because there are not enough transmitters to utilize the k-MPR ability, the network capacity cannot be further improved compared with the case when $k = \Theta(n)$. Therefore, the network capacity is $O\left(\frac{Wn}{\theta}\sqrt{\frac{m}{c}}\right)$ bit-m/s when $k = \Omega(n)$. □

Lemma 2. *The capacity of arbitrary (m, c, θ, k) wireless networks is $O\left(\frac{Wmn}{c}\right)$ bit-m/s.*

Proof. Consider the interface constraint of an arbitrary (m, c, θ, k) wireless networks. Because every node has m interfaces, there are interfaces in the whole network. Each interface can support at most $\frac{W}{c}$ bits/s, and the maximum distance that a bit can travel is $\Theta(1)$ in the network. Thus, the interface bound of the network is $O\left(\frac{Wmn}{c}\right)$ bit-m/s. □

According to Lemma 1, 2, we can obtain the network capacity that is $O\left(MIN_O\left(\frac{W}{\theta}\sqrt{\frac{kmn}{c}}, \frac{Wn}{\theta}\sqrt{\frac{m}{c}}, \frac{Wmn}{c}\right)\right)$ bit-m/s. The minimum bound of them is an upper bound on the network capacity. Then, we have the following theorem on the capacity of an arbitrary (m, c, θ, k) wireless network.

Theorem 1. *The upper bound on the capacity of an arbitrary (m, c, θ, k) wireless network is shown as follows:*

(1) When $\frac{kc}{m} = O(n\theta^2)$, network capacity is $O\left(\frac{W}{\theta}\sqrt{\frac{kmn}{c}}\right)$ bit-m/s.
(2) When $\frac{kc}{m} = \Omega(n\theta^2)$ network capacity is $O\left(\frac{Wmn}{c}\right)$ bit-m/s.

4.2. Constructive lower bound

Lemma 3. *Supposing $m, c, \overline{c}$ are positive integers such that $\overline{c} = \frac{c}{m}$. Then, the capacity of (m, c, θ, k) wireless networks is at least the capacity of $(1, \overline{c}, \theta, k)$ wireless networks.*

Proof. Consider a (m, c, θ, k) wireless network. We group the c channels into $\overline{c}$ groups (numbered from 1 to $\overline{c}$), with m channels per group. Specifically, channel group i, $1 \le i \le \overline{c}$, contains all channels j such that $(i-1)m + 1 \le j \le im$. Besides, nodes' antenna beams are aimed to proper directions to guarantee correct transmissions in all groups. The same transmission groups can be constructed as in $(1, \overline{c}, \theta, k)$ wireless networks to simulate this behavior in (m, c, θ, k) wireless networks. For the nodes of a group, we assign the m interfaces of each node to the m channels in the channel group i. The aggregated data rate of each channel group is $\frac{Wm}{c} = \frac{W}{\overline{c}}$. Therefore, a channel group in a (m, c, θ, k) network can support the same data rate as a channel in the $(1, \overline{c}, \theta, k)$ network. This means that the (m, c, θ, k) network can mimic the behavior $(1, \overline{c}, \theta, k)$ network. Hence, the capacity of (m, c, θ, k) wireless networks is at least the capacity of $(1, \overline{c}, \theta, k)$ wireless networks. □

Lemma 4. *Supposing that m and c are positive integers and the capacity of a (m, c, θ, k) wireless network is at least half of the capacity supported by a $\left(1, \lfloor\frac{c}{m}\rfloor, \theta, k\right)$ network.*

Proof. When $\lfloor\frac{c}{m}\rfloor = \frac{c}{m}$, the result directly follows from Lemma 3. Otherwise, $m < c$ and we use $c_i = m\lfloor\frac{c}{m}\rfloor$ of the channels in the (m, c, θ, k) network and ignore the rest of the channels. This can be viewed as a (m, c_i, θ, k) network with a total data rate of $W_i = \frac{c_i}{c}W$ bits/s. By Lemma 3, a (m, c_i, θ, k) network with total data rate of W_i can support at least the capacity of a $\left(1, \frac{c_i}{m}, \theta, k\right)$ network that is equal to the capacity of a $\left(1, \lfloor\frac{c}{m}\rfloor, \theta, k\right)$ network. However, when $W_i < W$, the (m, c_i, θ, k) network with total data rate W_i can achieve only a fraction $\frac{W_i}{W}$ of the capacity of a $\left(1, \lfloor\frac{c}{m}\rfloor, \theta, k\right)$ network with total data rate W. We have

$$\begin{aligned}\frac{W_i}{W} &= \frac{m}{c}\lfloor\frac{c}{m}\rfloor = \frac{\lfloor\frac{c}{m}\rfloor}{\frac{c}{m}}\\ &\ge \frac{\lfloor\frac{c}{m}\rfloor}{\lfloor\frac{c}{m}\rfloor + 1}, \text{since } \frac{c}{m} \le \lfloor\frac{c}{m}\rfloor + 1\\ &\ge \frac{1}{2}, \text{since } \lfloor\frac{c}{m}\rfloor \ge 1.\end{aligned}$$

Hence, a (m, c_i, θ, k) network can support at least half of the capacity supported by a $\left(1, \lfloor\frac{c}{m}\rfloor, \theta, k\right)$ network. Recall that $c_i \le c$, the capacity of (m, c, θ, k) wireless networks is at least the capacity of (m, c_i, θ, k) wireless networks. Therefore, we can obtain that a (m, c, θ, k) network can support at least half of the capacity supported by a $\left(1, \lfloor\frac{c}{m}\rfloor, \theta, k\right)$ network. This implies that asymptotically, a (m, c, θ, k) network has the same order of capacity as a $\left(1, \lfloor\frac{c}{m}\rfloor, \theta, k\right)$ network. □

Theorem 2. *The achievable capacity of an arbitrary (m, c, θ, k) wireless networks is bounded as follows:*

(1) When $\frac{kc}{m} = O(n\theta^2)$, network capacity is $\Omega\left(\frac{W}{\theta}\sqrt{\frac{kmn}{c}}\right)$ bit-m/s.
(2) When $\frac{kc}{m} = \Omega(n\theta^2)$ network capacity is $\Omega\left(\frac{Wmn}{c}\right)$ bit-m/s.

Proof. The unit-area plane is divided into a number of equal-sized rhombuses, whose length of sides is l. In each rhombus, there are eight positions for placing nodes as shown in Figure 10. Nodes at from T_1 to T_4 act as transmitters, and nodes from R_1 to R_4 act as receivers. T_3 and R_1 are at the distance of Δr from the center of the rhombus. In each rhombus, there are four position pairs to represent the data flow, from $T_1 - R_1$ to $T_4 - R_4$, and each pair can form a subrhombus whose side is l_0. Subrhombus represents DAs interference zone, and the beamwidth of all transmitters and receivers is θ.

Consider k-MPR ability, each receiver has k transmitters simultaneously. Hence, we place kg nodes at each of the transmitter positions and g nodes at receiver positions. Let $g = min\left(\frac{2\pi c_0}{\theta}, \frac{n\theta}{8(k+1)\pi}\right)$, where $c_0 = \lfloor\frac{c}{m}\rfloor$. Partition the square area into $\frac{n\theta}{8\pi(k+1)g}$ equal-sized rhombus cells, and place $\frac{8\pi(k+1)g}{\theta}$ nodes in each cell. Because the total area is 1, each cell has an area of $\frac{8\pi(k+1)g}{n\theta}$, and the length of side $l = \sqrt{\frac{8\pi(k+1)g}{n\theta\sin\theta}}$.

In each subrhombus, the distance from T_i to R_i is r, and each node is equipped with DAs with beamwidth θ. Hence, we have the side of subrhombus $l_0 = \frac{r\sqrt{2(1-\cos\theta)}}{2\sin\theta}$.

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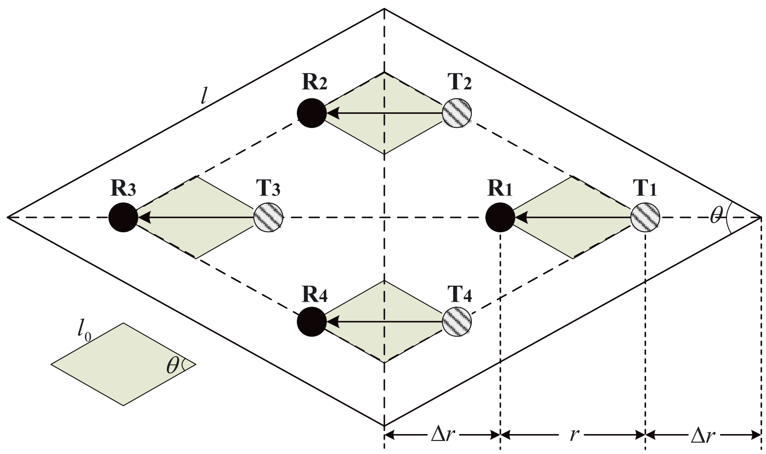

Figure 10. The placement of nodes within a cell. There are *g* nodes at each of the transmitter positions, and *kg* nodes at receiver positions.

Because of triangles and patchwork, we have $l = 2(1+2\Delta)l_0 = \frac{(1+2\Delta)r\sqrt{2(1-\cos\theta)}}{\sin\theta}$. Recall that $l = \sqrt{\frac{2\pi(k+1)g}{n\theta\sin\theta}}$, the transmission range can be obtained as

$$r = \frac{1}{1+2\Delta}\sqrt{\frac{4\pi(k+1)g\sin\theta}{n\theta(1-\cos\theta)}}. \tag{11}$$

Consider one node placed at any position from R_1 to R_4, namely n_R^1, n_R^2, n_R^3, and n_R^4, respectively. Besides, k nodes are placed at any position from T_1 to T_4, and we group those nodes into four groups by position, namely $group_T^1$, $group_T^2$, $group_T^3$, and $group_T^4$, respectively. There are k nodes in each group for transmitting. Under the protocol model of interference, it is easy to find that the distance from n_T^1 to $group_T^2$, $group_T^3$, and $group_T^4$ all are greater than $(1+\Delta)r$; therefore, n_R^1 can receive the packets from k nodes in the $group_T^1$ successfully. Similarly, n_R^i can receive the packets from k nodes in the $group_T^i$, $1 \le i \le \frac{n\theta}{32\pi(k+1)g}$, in the whole unit-area plane.

From the aforementioned construction, there are at most $\frac{nk}{k+1}$ concurrent transmissions within the domain. Restricting attention to only these transmissions, there are totally $\frac{nk}{k+1}$ simultaneous transmissions, each of which travels r meters and the interface of each node transmits at $\frac{W}{c_0}$ bits/s. By equation (11), this achieves the capacity of $\frac{1}{1+2\Delta}\sqrt{\frac{4\pi(k+1)g\sin\theta}{n\theta(1-\cos\theta)}}\frac{Wnk}{c_0(k+1)}$ bit-m/s. Recall that $g = min\left(\frac{2\pi c_0}{\theta}, \frac{n\theta}{8(k+1)\pi}\right)$. Hence, the capacity of the $(1, c_0, \theta, k)$ network is

$$\frac{2\pi}{1+2\Delta}\sqrt{\frac{2c_0(k+1)\sin\theta}{n(1-\cos\theta)}}\frac{Wnk}{c_0(k+1)\theta}, \text{if } g = \frac{2\pi c_0}{\theta};$$

$$\text{or } \frac{1}{1+2\Delta}\sqrt{\frac{\sin\theta}{2(1-\cos\theta)}}\frac{Wnk}{c_0(k+1)}, \text{if } g = \frac{n\theta}{8(k+1)\pi}.$$

Hence, the capacity of an arbitrary $(1, c_0, \theta, k)$ network is bounded by $\Omega\left(MIN_O\left(\frac{W}{\theta}\sqrt{\frac{kn}{c_0}}, \frac{Wn}{c_0}\right)\right)$ bit-m/s. According to Lemma 4, we have the capacity of an arbitrary (m, c, θ, k) to be $\Omega\left(MIN_O\left(\frac{W}{\theta}\sqrt{\frac{kmn}{c}}, \frac{Wmn}{c}\right)\right)$ bit-m/s, which leads to the aforementioned theorem. □

5. CAPACITY OF RANDOM NETWORKS

5.1. Upper bound

Different from arbitrary networks, the capacity of random networks is affected by three major factors [10]: network connectivity, interference, and destination bottleneck.

A. Connectivity constraint

Lemma 5. *The number of simultaneous transmissions on any particular subchannel is no more than* $\frac{16\pi}{c_1\theta^2\Delta^2r^2(n)}$ *in the protocol model.*

Proof. Supposing that node X_i in Figure 11 transmits successfully to node X_j on the same subchannel. Then, no other node X_k within a distance $\Delta r(n)$ of X_i can simultaneously receive a separate transmission on the same subchannel because of the protocol interference model and the triangle inequality.

Therefore, disks of radius $\frac{\Delta r(n)}{2}$ centered at each receiver on the same subchannel are disjoint. Because the area of each disk is $\frac{\pi\Delta^2r^2(n)}{4}$ and each receiver can receive from k transmitters simultaneously, it follows that the network can support no more than $\frac{4k}{c_1\pi\Delta^2r^2(n)}$ simultaneous

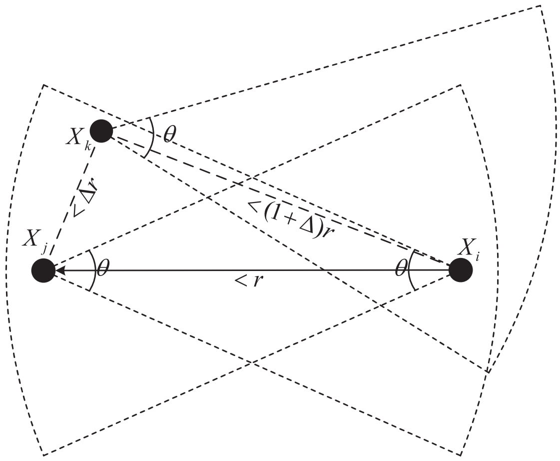

Figure 11. X_k cannot receive at the same time as X_j on the same subchannel.

transmissions on the same subchannel. Noting that antenna transmission and reception are both directional, the area of conditional interference zone is $\left(\frac{\theta}{2\pi}\right)^2 \pi r^2(n)$. Thus, the number of simultaneous transmissions on any particular subchannel should not exceed $\frac{4k}{c_1 \pi \Delta^2 r^2(n)} \left(\frac{2\pi}{\theta}\right)^2$. □

According to Lemma 5, each transmission over each subchannel is $\frac{W}{c}$ bits/s. By adding all the transmissions taking place at the time over all channels, we see that totally transmission rate cannot be more than $\frac{16\pi k}{\theta^2 \Delta^2 r^2(n)} W$ bits/s in the protocol interference model.

Recall that $\overline{L}$ denotes the mean length of a line connecting two independently and uniformly distributed point on the unit-area disk. Then, the mean number of hops taken by a packet is at least $\frac{\overline{L}}{r(n)}$. Because each source generates $\lambda(n)$ bits/s, there are n sources, and each bit needs to be relayed by on the average $\frac{\overline{L}}{r(n)}$ nodes, it follows that the total number of bits/s served by the entire network needs to be at least $\frac{\overline{L} n \lambda(n)}{r(n)}$.

The condition $\frac{\overline{L} n \lambda(n)}{r(n)} \leq \frac{16\pi k}{\theta^2 \Delta^2 r^2(n)} W$ will be needed to ensure that all the required traffic is carried.

Thus, $\lambda(n) \leq \frac{16\pi W k}{\overline{L} n \theta^2 \Delta^2 r(n)}$.

In order to know the range of $\lambda(n)$, we should know the range of $r(n)$ to guarantee the connectivity. According to the result in [18], we can take the transmission range $r(n) \geq \sqrt{\frac{\log n + 2k \log\log n}{\pi n}}$ to satisfy this requirement with high probability. Then, we can get the following upper bounds for this constraint.

(1) When $k = O\left(\frac{\log n}{\log\log n}\right)$, $n\lambda(n)$ is $O\left(\frac{Wk}{\theta^2}\sqrt{\frac{n}{\log n}}\right)$ bits/s.
(2) When $k = \Omega\left(\frac{\log n}{\log\log n}\right)$ and $k = O(n)$, $n\lambda(n)$ is $O(\frac{W}{\theta^2}\sqrt{\frac{nk}{\log\log n}})$ bits/s.
(3) When $k = \Omega(n)$, $n\lambda(n)$ is $O\left(\frac{Wn}{\theta^2\sqrt{\log\log n}}\right)$ bits/s.

B. General constraint

A random (m, c, θ, k) wireless network is a special case of arbitrary (m, c, θ, k) wireless networks, so the upper bound for the capacity of arbitrary (m, c, θ, k) wireless networks is applicable to random networks. In a random network, the distance between any source-destination pair is $\Theta(1)$ meter. Hence, according to the upper bound for arbitrary networks, the capacity of random networks is bounded as follows:

(1) When $\frac{kc}{m} = O(n\theta^2)$, $n\lambda(n)$ is $O\left(\frac{W}{\theta}\sqrt{\frac{kmn}{c}}\right)$ bits/s.
(2) When $\frac{kc}{m} = \Omega(n\theta^2)$, $n\lambda(n)$ is $O\left(\frac{Wmn}{c}\right)$ bits/s.

C. Destination bottleneck constraint

Consider a node X in a random (m, c, θ, k) wireless network. X has at most $D(n)$ flows whose destinations are X, and the maximum number of $D(n)$ is found in [10]. Under the network model we described earlier, each channel supports a data rate at $\frac{W}{c}$ bits/s. Because of k-MPR ability, the total data rate at which X can receive over m interfaces is $\frac{Wmk}{c}$. Moreover, because X should accommodate at least $D(n)$ flows, the data rate of the minimum rate flow is at most $\frac{Wmk}{cD(n)}$ bits/s. Therefore, by definition of $\lambda(n)$, $\lambda(n) \leq \frac{Wmk}{cD(n)}$, the network capacity $n\lambda(n) = O\left(\frac{Wmnk}{cD(n)}\right)$. According to Lemma 7, the network capacity is $O\left(\frac{Wmnk \log\log n}{c \log n}\right)$ bits/s.

Lemma 6. *The maximum number of flows for which a node in the network is a destination,* $D(n)$, *is* $\Theta\left(\frac{\log n}{\log\log n}\right)$, *with high probability.* [10]

By satisfying the three constraints previously mentioned, the upper bound for the network capacity presented in Section 3.3 is obtained.

5.2. Constructive lower bound

A routing scheme and a transmission scheduling scheme are constructed for any random $(1, c_a, \theta, k)$ wireless network, where $c_a = \lfloor \frac{c}{m} \rfloor$ to establish a tighter bound. According to Lemma 4, the achieved capacity is a lower bound for the capacity of random (m, c, θ, k) wireless networks.

5.2.1. Cell construction.

The torus of unit area is divided into square cells, and the area of each cell is $a(n)$, similar to the approach used in [19]. The size of each square denoted by $a(n)$ must satisfy the three constraints mentioned in Section 5.1.

It is found in [10] that when the size of each square is greater than a certain value, each square must contain a

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certain number of nodes. Therefore, it can guarantee successful transmissions from source nodes to destination nodes. Their lemma is stated here.

Lemma 7. *If $a(n)$ is greater than $\frac{50\log n}{n}$, each cell has $\Theta(na(n))$ nodes, with high probability.* [10]

We ensure that $a(n) \geq \frac{100\log n}{n}$ for a large n to simplify the calculation.

Consider the k-MPR ability, each node can receive k packets simultaneously on each of the c_a channels. In other words, we should guarantee that each node has at least kc_a neighbors. In addition, it is found in [7] that in a random network, the use of DAs at both the transmitter and the receiver can reduce the interfering area by $\left(\frac{\theta}{2\pi}\right)^2$, so the cell size should be large enough, $\left(\frac{\theta}{2\pi}\right)^2 a(n) \geq \frac{kc_a}{n}$. Thus, for a $(1, c_a, \theta, k)$ wireless network, $a(n)$ is equal to $max\left(\frac{100\log n}{n}, \frac{kc_a}{n}\left(\frac{2\pi}{\theta}\right)^2\right)$.

We take $(\frac{1}{D(n)})^2$ as another possible value for $a(n)$, where $D(n) = \Theta\left(\frac{\log n}{\log\log n}\right)$ (By lemma 7) to ensure the flow bottleneck constraint.

Then, we set $a(n) = min\left(max\left(\frac{100\log n}{n}, \frac{kc_a}{n}\left(\frac{2\pi}{\theta}\right)^2\right), \left(\frac{1}{D(n)}\right)^2\right)$.

It is found in [15] that the number of cells that interfere with any given cell is bounded by a constant $81(2+\Delta)^2\frac{\theta^2}{4\pi^2}$. In this paper, we would consider the impact of θ, so the following lemma can be gotten.

Lemma 8. *The number of cells that interfere with any given cell is bounded by $(\frac{\theta}{2\pi})^2 k_{inter}$(where $k_{inter} = 81(2+\Delta)^2$), which is independent of $a(n)$ and n.*

5.2.2. Routing scheme.

We construct a simple routing scheme that chooses a route with the shortest distance to forward packets. A straight line denoted by S-D line is passing through the cells where source node S and destination node D are located. Packets are delivered along the cells lying on the source-destination line. Then, we choose a node within each cell lying on the straight line to carry that flow. The node assignment is based on load balancing. In particular, the flow assignment procedure is divided into two steps.

Step 1: Source and destination nodes are assigned. For any flow that originates from a cell, source node S is assigned to the flow. Similarly, for any flow that terminates in a cell, destination node D is assigned to the flow. After this step, only those flows passing through a cell (not originating or terminating) are left.

Step 2: We assign the remaining flows. To balance the load, we assign each remaining flow to a node that has the least number of flows assigned to it. Thus, each node has nearly the same number of flows.

We present the following lemma to bound the number of source-destination lines that pass through any cell. This lemma has already been proved in [19].

Lemma 9. *The number of source-destination lines passing through any cell (including lines originating and terminating in the cell) is $O(na(n))$ with high probability.*[19]

Because $a(n)$ is greater than $\frac{100\log n}{n}$, each cell has $\Theta na(n)$ nodes (by Lemma 8). We can conclude that each intermediate node serves $O\left(\frac{1}{\sqrt{a(n)}}\right)$ flows. Each node is the originator of one flow, and each node is the destination of at most $D(n)$ flows, where $D(n)$ is $\Theta\left(\frac{\log n}{\log\log n}\right)$. Therefore, the total flows assigned to any node is at most $O\left(1 + D(n) + \frac{1}{\sqrt{a(n)}}\right)$. Recall that $a(n) = min\left(max\left(\frac{100\log n}{n}, \frac{kc_a}{n}\left(\frac{2\pi}{\theta}\right)^2\right), \left(\frac{1}{D(n)}\right)^2\right)$, the total flows assigned to any node is equal to $O\left(\frac{1}{\sqrt{a(n)}}\right)$ flows.

5.2.3. Scheduling transmissions.

A two-layer schedule is built as the same as [10]; however, we modified the second layer to satisfy the k-MPR ability and DAs. In the schedule model of this paper, any transmission in this network must satisfy the following two additional constraints simultaneously: (i) each interface only allows one transmission/reception at the same time; (ii) any two transmission groups (we group k transmissions received by a receiver into one transmission group) on any channel should not interfere with each other.

We describe the two-layer time slots as follows to satisfy the aforementioned two constraints, respectively.

First layer: In this layer, a second is divided into a number of edge-color slots, and at most, one transmission/reception is scheduled at every node during each edge-color slot. Hence, the first constraint is satisfied. We construct a routing graph in which vertices are the nodes in the network and an edge denotes transmission/reception of a node. In [10], it shows that this routing graph can be edge-colored with at most $O\left(\frac{1}{\sqrt{a(n)}}\right)$ colors. Therefore, we divide one second into $O\left(\frac{1}{\sqrt{a(n)}}\right)$ edge-color slots and each slot has a length of $\Omega\left(\sqrt{a(n)}\right)$ seconds. Each slot is stained with a unique edge-color. Because all edges connecting to a vertex use different colors, each node has at most one transmission/reception scheduled in any edge-color time slot.

Second layer: In this layer, each edge-color slot can be further split into smaller mini-slots and satisfies both the two aforementioned constraints. We construct an interference graph in which vertices are the transmission groups in the network and edges denote interference between two groups. According to Lemma 9, every cell has at most a constant number of interfering cells with a factor $\left(\frac{\theta}{2\pi}\right)^2$, and each cell has $\Theta(na(n))$ nodes (by Lemma 8). Thus, each node has at most $O\left(\left(\frac{\theta}{2\pi}\right)^2 \frac{na(n)}{k}\right)$ transmission groups' edges in the interference graph. It is well-known that a graph with maximum degree v can be vertex-colored with at most $v+1$ colors [20]. Hence, the interference graph can be vertex-colored with at most $O\left(\left(\frac{\theta}{2\pi}\right)^2 na(n)\right)$ colors. Therefore, we use $k_1\left(\frac{\theta}{2\pi}\right)^2 \frac{na(n)}{k}$ to denote the number of vertex-colors (where k_1 is a constant). Two transmission groups assigned the same vertex-color that do not interfere with each other, whereas two groups stained with different colors may interfere with each other. So, we need to schedule the interfering groups either on different channels or at different mini-slots on the same channel. Thus, we divide each edge-color slot into $\left\lceil\left(\frac{\theta}{2\pi}\right)^2 \frac{k_1 na(n)}{kc_a}\right\rceil$ mini-slots on every channel. Recall that each edge-color slot has a length of $\Omega(\sqrt{a(n)})$ seconds, so each mini-slot has a length of $\Omega\left(\frac{\sqrt{a(n)}}{\left\lceil\left(\frac{\theta}{2\pi}\right)^2 \frac{k_1 na(n)}{kc_a}\right\rceil}\right)$ seconds.

As shown in Figure 12, channels are numbered from 1 to c. First, one second is divided into $O\left(\frac{1}{\sqrt{a(n)}}\right)$ edge-color slots on every channel to satisfy constraint (1). Second, each edge-color slot is further divided into $\left\lceil(\frac{\theta}{2\pi})^2 \frac{k_1 na(n)}{kc_a}\right\rceil$ mini-slots. In every mini-slot, we can guarantee that every transmission satisfies the two constraints simultaneously.

5.2.4. Utilization of k-MPR ability.

There are at most k concurrent transmissions within its receiving range for each receiver. However, the number of concurrent transmissions for a receiver is less than k in some cases. If k transmissions are received by a receiver simultaneously, at least k nodes, being transmitters, will interfere with the receiver. Besides, in the random network, the number of flows served by each node should be greater than k. In other words, each node has the ability of serving k flows to the least extent. We should satisfy the following two fully utilization requirements (FUR) simultaneously to fully utilize k-MPR ability.

FUR(1) The number of concurrent receptions cannot exceed the number of nodes which are located at the interfere cells of the receiver $\left(k = O\left(\frac{\theta^2 k_{inter} na(n)}{c_a}\right)\right)$.

FUR(2) The number of concurrent receptions cannot exceed the number of flows each node serves $\left(k = O\left(\frac{1}{\sqrt{a(n)}}\right)\right)$.

5.2.5. The achieved capacity lower bound.

We first analyze the capacity of the $(1, c_a, \theta, k)$ wireless networks and then extend the results to a (m, c, θ, k) wireless network. In the $(1, c_a, \theta, k)$ wireless network, recall that each mini-slot has a length of $\Omega\left(\frac{\sqrt{a(n)}}{\left\lceil\left(\frac{\theta}{2\pi}\right)^2 \frac{k_1 na(n)}{kc_a}\right\rceil}\right)$ seconds, and each channel can transmit at the rate of $\frac{W}{c_a}$ bits/s. Therefore, $\lambda(n) = \Omega\left(\frac{W\sqrt{a(n)}}{c_a\left\lceil\left(\frac{\theta}{2\pi}\right)^2 \frac{k_1 na(n)}{kc_a}\right\rceil}\right)$ bits can be transported in each mini-slot on every channel.

Because $\left\lceil\left(\frac{\theta}{2\pi}\right)^2 \frac{k_1 na(n)}{kc_a}\right\rceil \le \left(\frac{\theta}{2\pi}\right)^2 \frac{k_1 na(n)}{kc_a} + 1$, we have $\lambda(n) = \Omega\left(\frac{W\sqrt{a(n)}}{c_a\left(\frac{\theta}{2\pi}\right)^2 \frac{k_1 na(n)}{kc_a} + c_a}\right)$ bits/s. Hence, $\lambda(n) = \Omega\left(MIN_O\left(\frac{Wk}{\left(\frac{\theta}{2\pi}\right)^2 n\sqrt{a(n)}}, \frac{W\sqrt{a(n)}}{c_a}\right)\right)$ bits/s.

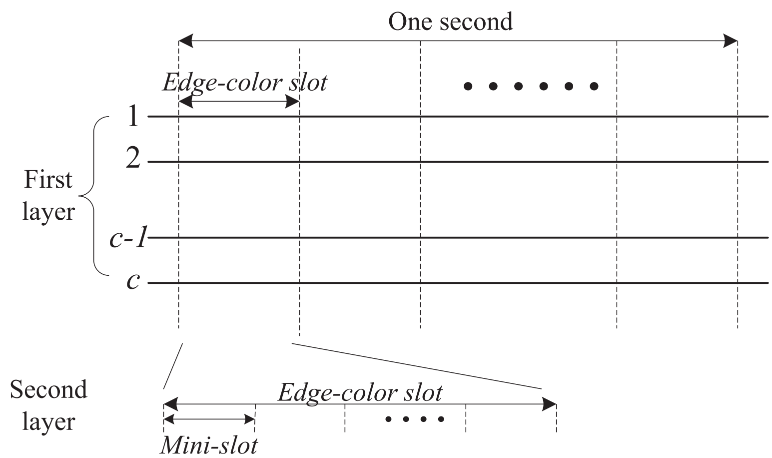

Figure 12. Two-layer transmission schedule.

Table I. The capacity gain over (m, c, θ, k) networks for random networks.

Capacity gain	*c/m-small*	*c/m-moderate*	*c/m-large*
k-small	A $\left(\frac{2\pi}{\theta}\right)^2 k$	C $\left(\frac{2\pi}{\theta}\right)\sqrt{k}$	E No gain
k-moderate	B $\left(\frac{2\pi}{\theta}\right)^2 \sqrt{k}$	C $\left(\frac{2\pi}{\theta}\right)\sqrt{k}$	E No gain
k-large	D $\left(\frac{2\pi}{\theta}\right)^2$		E No gain

Recall that each flow is scheduled in one mini-slot on each hop during one-second interval and every source-destination flow can support a per-node throughput of $\lambda(n)$ bits/s. Therefore, when k-MPR is fully utilized, the total network capacity is $n\lambda(n)$ which is equal to $\Omega\left(MIN_O\left(\frac{Wk}{\left(\frac{\theta}{2\pi}\right)^2\sqrt{a(n)}}, \frac{Wn\sqrt{a(n)}}{c_a}\right)\right)$ bits/s. In a (m, c, θ, k) wireless network, by Lemma 4, when k-MPR is fully utilized, the total network capacity is equal to $\Omega\left(MIN_O\left(\frac{Wk}{\left(\frac{\theta}{2\pi}\right)^2\sqrt{a(n)}}, \frac{Wmn\sqrt{a(n)}}{c}\right)\right)$ bits/s.

Lemma 10. *Assume $k_1 = O(k_2)$. C_1 is the capacity of (m, c, θ, k_1) wireless networks, and C_2 is the capacity of (m, c, θ, k_2) wireless networks, then $C_1 = O(C_2)$.*

Proof. The (m, c, θ, k_2) wireless networks can imitate (m, c, θ, k_1) wireless networks by restricting k_2 to k_1. In addition, the capacity of (m, c, θ, k) wireless networks will not decrease with the increase of k. Therefore, the capacity of (m, c, θ, k_2) wireless networks is at least the capacity of (m, c, θ, k_1) wireless networks. Hence, we get the lemma. □

According to FUR (1) and (2), we can know whether k-MPR can be fully utilized. Recall that $a(n) = min\left(max\left(\frac{100\log n}{n}, \frac{kc_a}{n}\left(\frac{2\pi}{\theta}\right)^2\right), \left(\frac{1}{D(n)}\right)^2\right)$, where $D(n) = \Theta\left(\frac{\log n}{\log\log n}\right)$. When fully utilizing k-MPR ability, we can get the lower bound through substituting for the three values. Moreover, when k-MPR ability is not fully utilized, we can apply Lemma 10 to get the lower bound. The lower bound for the random network capacity presented in Section 3.3 is obtained.

6. DISCUSSION

The arbitrary network bounds may be viewed as the best case bounds on network capacity, because the location of nodes and traffic patterns can be controlled in the arbitrary network and it is more realistic on network conditions. From the results of this work, when $\frac{kc}{m} = O\left(\left(\frac{\theta}{2\pi}^2\right)n\right)$, a (m, c, θ, k) wireless network can achieve a capacity gain $\frac{2\pi}{\theta}\sqrt{k}$ over a $(m, c, 2\pi, 1)$ wireless network. However, when $\frac{kc}{m}$ is too large, the MCMI network with MPR and DAs has no capacity gain over a $(m, c, 2\pi, 1)$ wireless network. The value of k and m are based on the ability of hardware and cost; the number of non-overlapping channels c would be smaller than $\left(\frac{\theta}{2\pi}\right)^2 \frac{mn}{k}$ to get a greatly capacity gain.

Recall the results for random networks, there are three ranges of k; we abstractly use $k\text{-}small$, $k\text{-}moderate$, and $k\text{-}large$ to denote the range of k. When ratio $\frac{c}{m}$ is sufficiently small in each range, a (m, c, θ, k) wireless network can achieve a capacity gain at most $\left(\frac{\theta}{2\pi}\right)^2 k$ over a $(m, c, 2\pi, 1)$ wireless network. Moreover, we use $c/m\text{-}small$, $c/m\text{-}moderate$, and $c/m\text{-}large$ to denote the range of $\frac{c}{m}$. We can conclude that a (m, c, θ, k) wireless network has a capacity gain over a $(m, c, 2\pi, 1)$ wireless network as shown in Table I. The region-A has the greatest capacity gain $\left(\frac{2\pi}{\theta}\right)^2 k$ and the next is the region-B $\left(\frac{2\pi}{\theta}\right)^2\sqrt{k}$, the region-C $\left(\frac{2\pi}{\theta}\right)\sqrt{k}$, and the region-D $\left(\frac{2\pi}{\theta}\right)^2$. When $\frac{c}{m}$ is $c/m\text{-}large$, there is no capacity gain over a $(m, c, 2\pi, 1)$ wireless network (region-E).

7. CONCLUSION

This paper considered a wireless network that integrates MCMI, MPR, and DA. It studied the capacity of (m, c, θ, k) wireless networks under both arbitrary and random networks. The results show that using DAs and MPR in MCMI networks cannot only enhance network connectivity but also mitigates interferences. However, when $\frac{kc}{m}$ is large enough in the arbitrary networks, there is no capacity gain over using omni-DAs and no-MPR. In the random networks, when $\frac{c}{m}$ is sufficiently large, there is no gain. It also shows that combining MCMI with k-MPR and DAs can achieve at most $\frac{2\pi}{\theta}\sqrt{k}$ capacity gain in arbitrary networks compared with $(m, c, 2\pi, 1)$ wireless networks and $(\frac{2\pi}{\theta})^2 k$ capacity gain in random networks.

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